Acknowledgments

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Base Pressure on Sharp and Blunt Conical Bodies at Supersonic Speeds

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Nomenclature

B	= nose bluntness ratio, d_n/d_B
d_B	= cone base diameter
d_n	= cone nose diameter
M_B	= blunt cone surface Mach number just ahead of the base, inviscid
M_{eff}	= effective Mach number of flow into base region, Eq. (1)
M_S	= sharp cone surface Mach number, inviscid
M_∞	= freestream Mach number
M'_{eff}	= effective Mach number, M_{eff}, isentropically expanded through cone semiangle (back to parallel)
P_B	= cone base pressure
P_c	= cone surface pressure just ahead of the base
P_∞	= freestream pressure
P'_c	= cone surface pressure, P_c, isentropically expanded through cone semiangle (back to parallel)
θ_c	= cone semiangle, deg

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Introduction

THE base drag of a conical body in supersonic flight represents a large fraction of the total vehicle drag. Twenty to sixty percent of the total drag can be associated with the base flow dependent on cone geometry, Mach number, and Reynolds number. Since the base drag fraction is so large and the difference between the base and freestream pressures can be small, a very precise assessment of base pressure is required if an accurate measurement of total drag is sought. With regard to wind tunnel testing, the test Reynolds number, sting geometry, and bow shock wave reflection from the tunnel wall must be carefully controlled to obtain the required accuracy.

A recent study at AEDC has produced a simple, but very accurate, correlation of the cone base pressure in fully turbulent flow at Mach numbers of 1.5 to 8. The fully turbulent base pressure data used in the correlation have been obtained in the supersonic tunnels at AEDC over the past two decades. The correlation is presented to permit an accurate prediction of supersonic base drag which can then be coupled with an analytically or experimentally determined forebody drag to predict total drag.

Correlation

Several early investigators[1-3] detected that the base pressure on various bodies could be correlated by using the local flow condition just ahead of the base as the nondimensionalizing term. Whitfield and Potter[3] found that the base pressure in a sharp cone flowfield yielded an acceptable correlation curve by using the local cone static pressure, p_c, and local cone Mach number isentropically expanded through the cone semiangle (back to parallel flow). In addition, the effect of bluntness on the base pressure could be accounted for if an "effective" local Mach number were used in the correlation. Comparing sharp nose to 0.3 bluntness cone data, a simple average of the sharp cone surface Mach number and blunt cone surface Mach number for any given cone angle and freestream Mach number appeared to be a suitable "effective" Mach number for the blunt body flow. With more extensive data since the work of Ref. 3, an improved form of an effective Mach number for blunt conical bodies has been established. This effective Mach number takes the form

$$M_{\text{eff}} = M_S - (M_\infty/3)B(M_S - M_B) \tag{1}$$

In this expression, the bluntness ratio, B, and freestream Mach number are seen to weight the effective Mach number closer to the sharp cone surface value, M_S, or the blunt cone surface value, M_B, than a simple average. Physically, the expression may be explained with the aid of Fig. 1. Outside of the entropy layer, the sharp cone and blunt cone flow Mach numbers are identical for a given freestream condition and cone angle. Inside the entropy layer, the Mach numbers digress. The relative thickness of the entropy layer is dependent on the bluntness ratio and the thickness of the fluid layer influencing the wake. Hence, the inviscid surface Mach number M_B is not, in itself, representative of the properties of the total fluid layer which influence the wake for a blunt body and must be weighted with the other region flow. However, it can be shown that the variation in static pressure through this layer is small, such that the cone (either sharp or blunt) surface pressure is an adequate parameter. The inviscid cone surface properties used in Eq. (1) are obtained from existing computer flowfield solutions or table lookup.

This semiempirical technique has been found to correlate an extensive collection of AEDC/VKF turbulent base pressure data between Mach 2 and 8 for cone semiangles from 6 to 14 deg and bluntness ratios from sharp up to 0.7. In essence, the base pressure achieved on a conical body for fully turbulent flow is directly related to a weighted Mach number and the local surface pressure in the layer of fluid just ahead of the base when expanded back to the parallel. Base pressure

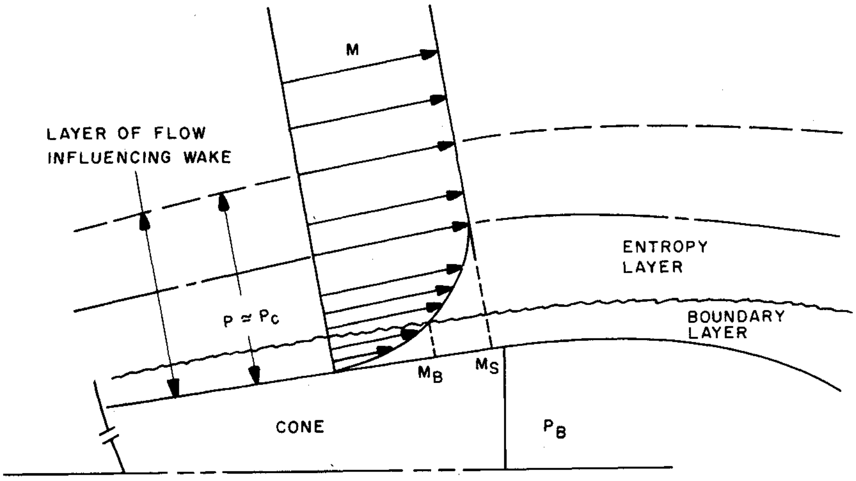

Fig. 1 Influence of body flow on eventual wake flow.

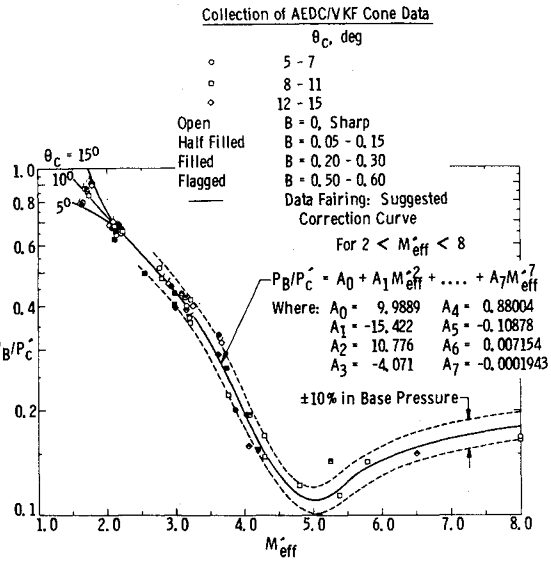

Fig. 2 Cone base pressure correlation.

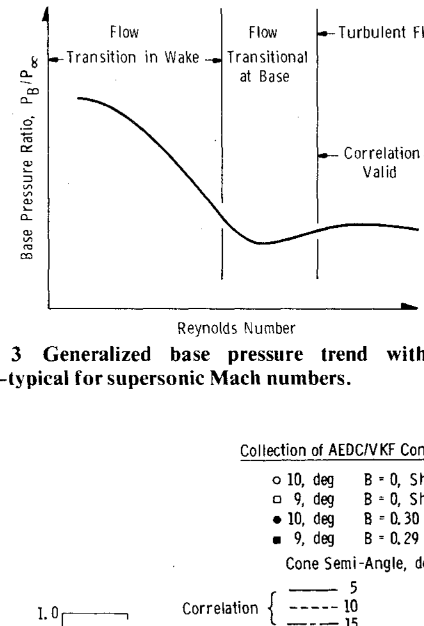

Fig. 3 Generalized base pressure trend with Reynolds number—typical for supersonic Mach numbers.

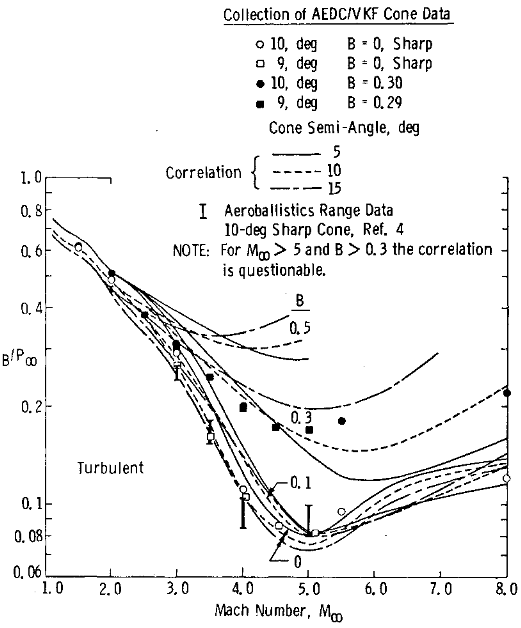

Fig. 4 Base pressure variation for a family of conical bodies—turbulent, interference-free flow.

results on cones are presented in the correlation terms in Fig. 2. A least squares polynominal fit to the correlation curve which is valid for $2 \leq M'_{eff} \leq 8$ is also given in Fig. 2. Previous base pressure data, with poorly established turbulence levels, tunnel wall interference in the model wake or oversized stings, were eliminated from the correlation data set. The flow condition for the data used in this correlation is felt to be fully turbulent at or ahead of the cone base as a result of Reynolds number surveys and trip sizing studies. Hence, the correlation is valid for the flow region at the right of Fig. 3. The bow wave which reflects from the tunnel wall intersected the wake at least 6 base diameters downstream of the model base for all of the test data used in the correlation (even at Mach 1.5). In addition, it appears that the stings used in all of the tests chosen for correlation, with diameters which did not exceed 0.3 d_B and unobstructed sting lengths of at least 4 d_B or greater, had no measurable influence on the cone base pressure as will be shown. All of the data for the very wide range of shapes are seen to fall within about ±10% of a mean correlation curve even though the data in uncorrelated terms vary by nearly an order of magnitude. This correlation, using weighted flow properties at the base, represents an improvement at supersonic speeds relative to previous correlations utilizing cone surface properties only. The turbulent base pressure derived from the correlation for nose bluntness ratio greater than 0.3 and Mach number greater than 5.0 is questionable (the predicted P_B/P_∞ being too great). An adjustment of the bluntness term in Eq. (1) could be made to improve this, however, ambiguities exist in available data for $M_\infty > 5$ and $B > 0.3$ and a slight refinement of Eq. (1) has not been attempted.

Prediction

The base pressure ratio for 5 to 15 deg semiangle cones, with sharp to 0.5 bluntness noses, is given in Fig. 4. At Mach numbers of two or less, only the cone angle is important; at about Mach 3, cone angle and bluntness ratio are equally important; and above Mach 3, bluntness is the relatively stronger parameter in establishing the base pressure level. The validity of the correlation is demonstrated by comparison with specific 9 and 10 deg cone data for sharp and 0.3 bluntness ratio noses which are given in Fig. 4. Data for other cone angles and bluntness ratios exhibit an equally good agreement with the prediction as would be expected based on Fig. 2; however, all of these data are not given in Fig. 4 for clarity.

Also, total drag measurements obtained in an aeroballistics range on sharp 10-deg semiangle cones during the course of the transition experiments reported in Ref. 4 were analyzed. A sting free, flight base pressure was extracted from these data by subtracting a calculated forebody drag (pressure and skin friction) from the measured total drag. This calculations can be made with great precision for the sharp slender cone in question since the location of transition was measured. The relatively precise free-flight base pressure results obtained in this manner are given in Fig. 4 and further confirm both the absence of sting effects and the validity of the correlation. Even at Mach numbers in excess of eight, base pressure predictions utilizing this technique are in good qualitative agreement with the available turbulent data for sharp or small bluntness cones (up to Mach ~ 20).

Summary

At supersonic speeds, the base drag of sharp or blunt conical bodies represents a significant fraction of the total configuration drag. Correlation parameters are presented which reduce fully turbulent cone base pressures to a single curve for a wide range of nose bluntness ratios and cone semiangles. The base pressure which is computed using this technique is within ±10% of a fully turbulent flight equivalent value or better.

Acknowledgments

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Technical Comments

Comment on "Flutter of Flat Finite Element Panels in a Supersonic Potential Flow"

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PROFESSOR T. Y. Yang of Purdue University has just published a new solution[1] to one of this writer's thesis problems. Since the thesis[2] was written at UCLA almost two decades ago, and since there has been no interest in the two-dimensional, potential flow, panel flutter problem since 1965, it seems appropriate to ask "What's new?"

The thesis was intended to resolve some unexpected differences between the traveling wave solution for an unsupported infinite panel by Miles[3] (the writer's advisor) and the standing wave solution for a periodically supported infinite panel by Hedgepeth, Budiansky, and Leonard[4] of the NACA Langley Research Center. The second part of the thesis considered a finite length panel with equally spaced supports as an approximation to the periodically supported infinite panel. Three configurations were analyzed: single-span panels with pinned and clamped ends, and a two-span panel continous over pinned supports. A finite element formulation of the flutter problem used structural and aerodynamic influence coefficients (AICs). This formulation was an extension of the matrix method of Duncan and Collar[5] for free vibration analysis to solve the flutter problem by adding the aerodynamic forces via AICs. This extension was published in 1956.[6] The thesis developed the AICs for the two-dimensional panels and these were published later in dimensionless form in a survey paper[7] on unsteady AICs.

The single-span solutions using influence coefficients were compared with solutions of Shen[8] and Nelson and Cunningham[9] by the Galerkin method. It was seen that Shen's two-mode solution had not converged and that Nelson and Cunningham did achieve convergence with four modes by comparison with the nine-degree-of-freedom influence coefficient solution, and the accuracy of the influence coefficient formulation was established. However, only limited computed results were obtained in the thesis. Stability boundaries were not obtained at the lower Mach numbers ($M = 1.3$ and $\sqrt{2}$) but were obtained at the higher Mach numbers ($M = 1.56$ and 2.0), and the stability boundaries obtained at the high Mach numbers for the two-span panel were not correct. The latter difficulty was pointed out in 1963 by Dowell[10] in his thesis at MIT using Piston Theory, and the two-span stability boundary was corrected in 1964.[11] The corrected results agreed closely with Dowell's results and demonstrated that Piston Theory was adequate for two-dimensional panels at Mach numbers as low as 1.56. In 1964, Lock and Farkas, using the influence coefficient method, set out to study the single-degree-of-freedom stability boundary at low Mach numbers. Their results were reported in 1965.[12] At that point in time, the two-dimensional, potential flow, panel flutter problem was well understood, and investigators turned their attention to the three-dimensional problems of plates and shells.

The foregoing historical account permits us to answer the question "What's new?". Reference 1 uses the stiffness representation of the structure where the earlier finite element studies used a flexibility representation. Today, the stiffness formulation is preferable; before 1965, when NASTRAN was just being launched with equal requirements for the stiffness and flexibility approaches, it did not seem to matter. Reference 1 then uses Archer's consistent mass matrix[13] where the earlier studies had used a diagonal mass matrix. Today, the consistent mass matrix is preferable, just as a coupled mass matrix was employed in Ref. 6. However, with

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